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Aeroelastic Instability in F100 Labyrinth Air Seals

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Analysis of an aeroelastically unstable F100 labyrinth air seal has shown a stable design could be produced by tailoring the knife-edge lip clearance. An extensive engine experimental program confirmed that seals experienced high cycle fatigue (HCF) cracking resulting from clearance. Correlation has been demonstrated between analytically predicted instability and that observed during test engine operation. Experimental apparatus was developed to selectively augment or squelch the instability. These techniques improve definition of stability boundaries and provide a conservative means of substantiating stability margin with the revised configuration.

Nomenclature

A = area of cavity cross section
E = number of waves around circumference
g = gravitational constant
h = lip clearance
i = imaginary unit
K = local slope of curve (always subscripted α or θ)
N = number of seal lips
P = total pressure
R = radius of seal
t = time
U = circumferential flow velocity
V = specific volume
W = mass flowrate
X = circumferential position
α = flow coefficient
γ = ratio of specific heats
Δ = prescript indicating amplitude of an oscillating quantity
ϕ = flow function
ρ = density
τ = thickness of a lip
Ω = rotational speed of vibrating structure
ω = frequency of vibration

Subscripts
d = downstream of last lip
n = lip or cavity number
s = relative to vibrating structure
u = upstream of first lip

Introduction

LABYRINTH seals are a common feature of turbine engines. They are used in annular cavities between the rotating and nonrotating structures to isolate high-pressure cavities from lower pressure cavities in which a nominal clearance must be maintained to accommodate tolerance accumulation and off-design point operation. The flow through the seal is reduced by repeatedly throttling across a series of lips and by dispersing the kinetic energy at each lip with a step in the flowpath. The leakage flow through the labyrinth seal is typically only 60% of the leakage which would occur across an unimproved gap with the same overall pressure ratio.

The objective of maximizing cycle efficiency requires that flow through the seal, or leakage, be minimized. The flow through the labyrinth seal is a function of the geometry of the seal and the engine airflow parameters on the seal inlet and discharge. The geometry of configuration parameters are: 1) the number of knife-edges, 2) radial offset step from land to land, 3) knife-edge axial spacing, 4) knife-edge radius, 5) the volume of the cavities between knife-edges, and 6) clearance between the knife-edges and the land.

The engine parameters influencing seal leakage are gas temperature and pressure of the upstream and downstream cavities and rotor speed.

Once the constraints of available space, weight and cost have been exercised, the remaining variable, which the designer can control to minimize seal leakage, is the clearance between the knife-edge and its land. This clearance is impacted by the relative rotor frame motion which occurs because of rotor vibratory whirl, maneuver deflections, thermal growth, and tolerance accumulations.

The factors controlling the steady-state airflow and physical configuration of the seal system also affect the dynamic vibratory characteristics of the seal system. Turbine engine history has shown numerous cases of labyrinth seal designs satisfying functional requirements but lacking satisfactory durability because of high cycle fatigue resulting from vibration excited by forces in the immediate environment.

Causes of Seal Vibration

Destructive levels of vibration can be excited in the following manners:

1) Mechanically—resonant response of the seal rotating or stationary members excited by local engine forces.

2) Acoustically—acoustic forces occurring as standing waves, spinning waves, or cavity resonances may coincide with a mechanical resonance in frequency or wave speed and mode shapes.

3) Aeroelastic instability—mechanical deflections produce a pressure perturbation that, in turn, reinforces the mechanical deflection. The interaction persists at a mechanical natural frequency which has a deflected shape (mode shape) most sympathetic with the aeroelastic coupling.

This discussion concentrates on the aeroelastic instability phenomena and relates a recent example confronted in the F100 turbofan engine.

Seal Application

The seal on which the aeroelastic instability encountered is at the high compressor discharge of the F100, a high thrust-to-weight turbofan engine (Fig. 1). The knife-edges are on the seal rotor, the land is stationary. The land surfaces are faced with an abradable material. The seal was located remote from

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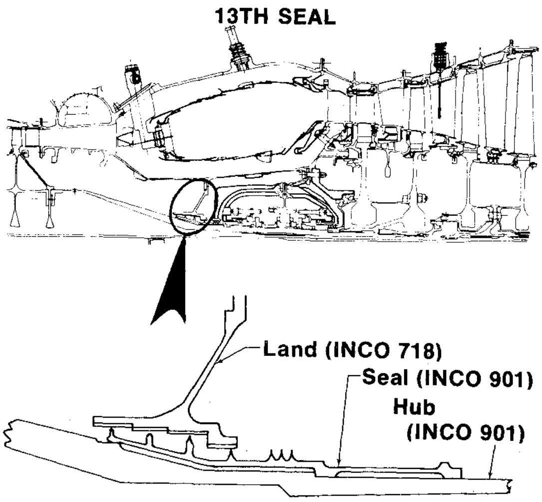

Fig. 1 Engine cross section.

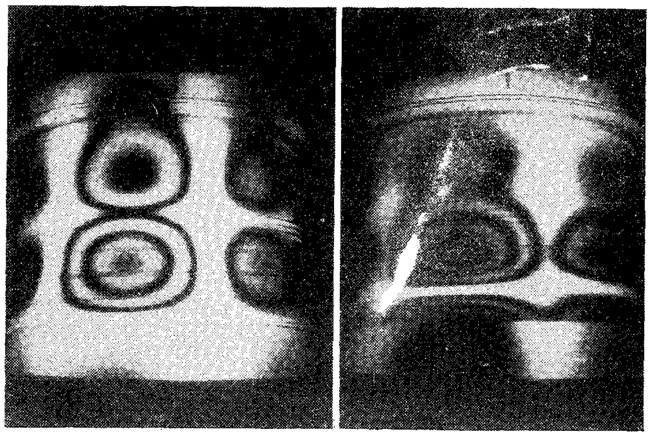

Fig. 2 Tight snap, loose snap.

the compressor discharge so the radius of the seal and land are relatively small to minimize leakage.

The knife-edged rotor was made to be removed and refurbished at minimum cost, so rather than being integral with the compressor rear hub, it is a thin conical shell secured to the hub radially by snaps and axially by a stack-up secured by the compressor aft bearing nut. The seal land is a conical forging attached to the i.d. of the engine burner case (Fig. 1).

Engine Operation Affects Vibration Characteristics

The vibratory characteristics of the thin knife-edge shell are normally dominated by the more massive compressor hub. An exception occurs during a maximum rotor speed transient from idle to maximum speed. The knife-edge shell is heated by compressor discharge air. The hub is cooled on the i.d. by bleed from a forward snap on the knife-edge shell to grow temporarily loose from the hub. During this brief interval, a maximum of 30 s, the fixity of the knife-edge shell is altered, permitting a unique family of vibratory shell modes to occur.

Historical criteria for avoiding aeroelastic instability emphasizes the importance of seal rigidity and attitude. In the configuration present during steady-state operation, the knife-edge platform was supported at both upstream and downstream flow boundaries. During the brief transient exposure to a loose front snap, the knife-edge platform is supported from the downstream side which also has been considered to be inherently stable. To satisfy an empirical stability requirement a weight penalty of approximately seven pounds would have been required to provide a shell stiffness matching the criteria minimum. Evaluation by a P&WA aeroelastic stability analysis indicated the seal system remained stable in the fundamental vibration modes. Both the steady-state, fore and aft supported, and the transient downstream supported systems had demonstrated stable operation in previous seal designs. The seal was designed to withstand moderate steady-state forces without additional stiffening.

Development History

The performance of this seal configuration had been excellent throughout development testing, endurance tests, and Accelerated Operational Mission Tests. Leakage goals were met, the clearance between the knife-edges and the land showed very moderate increases with operation, and there were no durability problems.

Subsequent to all development testing, high cycle fatigue cracks were discovered in the knife-edged seal rotor during scheduled maintenance inspections of operational production engines. The first cracked seal was detected after several hundred engines had been introduced to service. Although no failures occurred, subsequent inspections detected additional cracked seals.

Description of Cracks

The seals in which cracks were found fell into two general categories. The first category had radial cracks in the thin knife-edge members alone, and only in the first knife-edge; the second category indicated origins in the first and second knife-edges but some of the origins had advanced into the barrel of the shell and then propagated axially. Several cases were found in which the crack propagated forward from the second knife-edge joining a barrel crack propagating backward from the first knife-edge completely splitting the hoop at the forward first knife-edge. These axial craks had allowed enough knife-edge centrifugal growth to gouge out the mating abradable material on the land to a depth of 0.020 in. and to 0.030 in. radially.

Preliminary Evidence

Fractography and a comparative review of dimensional inspections, material properties, and the operational exposure of cracked seals vs equal or higher time uncracked parts were undertaken. The results were as follows:

1) The initiation and propagation mechanism had been high cycle fatigue.

2) Dimensional inspections found the cracked seals and their lands to be at the tighest side of the design clearance tolerances on one or more of the first four knife-edge gaps while the high-time uncracked parts tended to be on the most open side of the design clearance tolerances.

3) The fatigue properties of cracked seals were not worse than material from seals which had not cracked, or from virgin material.

4) The operational and mission exposure of engines with seals having cracks were not unique. The common aspects of the operation of engines with cracked seals were typical of the entire engine population.

5) Seals having axial barrel cracks had been discovered in engines with total accumulated operating time ranging from a low of 21 h to a high of 295 h.

6) No axial barrel cracks had propagated aft more than midway between the second and third knife-edges. No seals had axial cracks at more than one circumferential location.

Preliminary Assumptions

Preliminary assumptions were as follows:

1) Shell mode—The seal was cracking during vibration in a shell mode; i.e., mode shape with diametral nodes.

2) Stress profile—In the specific vibratory mode, stresses would have to be significantly higher in the first and second knife-edges than in the aft knife-edges.

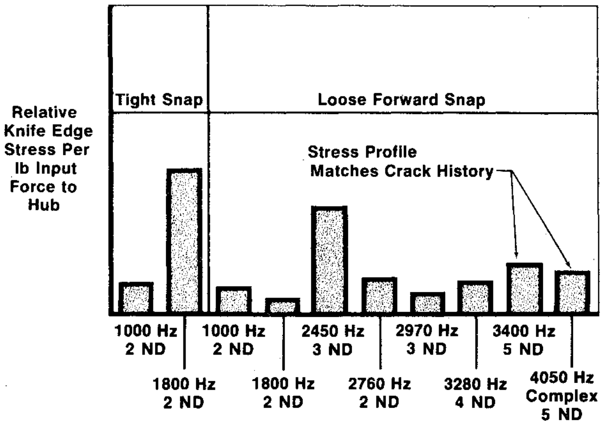

Fig. 3 Lab shaker test results.

3) Stress die-out—Some feature of the vibratory mode shape resulted in a low level of stress between the second and third knife-edges; this was the extreme point to which an HCF crack had propagated.

4) Self healing—The axial barrel cracks could propagate rapidly (21 h) but had never gone beyond the split-hoop phase and initiated additional axial cracks at another circumferential location.

5) Typical engine—Seal vibration could occur during typical operating conditions every engine would encounter.

6) Unique combination—Because the cracking was a selective not a general problem, it required a matched set of parts that were tuned by tolerance variations.

7) Common denominator—The uniqueness was related to the physical feature observed to be common in all of the same systems with cracked seals: tight knife-edge clearances between the seal rotor and the land.

Laboratory tests, analysis, and core engine tests were initiated to find a solution to the cracking problem.

Laboratory Tests

Laboratory tests determined that the fatigue properties of the material in the cracked seals were typical. Tests defined stress and deflection profile (mode shape), frequency, responsiveness, and damping in the vibration modes of the seal rotor and the seal land. Subassemblies were driven with electromagnetic and piezoelectric shakers to greater than 20,000 Hz. Stresses and deflections were measured and the vibratory motions visualized with holography.

Vibration tests on the land subassemblies were also conducted to determine if mechanical coincidence was the responsible mechanism. This phenomenon requires the rotating seal and its stationary land to have similar mode shapes and identical wave speeds.

The vibration tests of the seal rotor reflected the two extremes of fixity that occur: the normal steady-state condition when it is tightly snapped to the hub fore and aft; and the transient condition which occurs briefly after a rapid acceleration to maximum speed where forward snap grows loose from the hub due to a thermal gradient.

The seal rotor was responsive in only two modes during the tight forward snap condition. These were both 2-nodal diam modes which were controlled by the massive hub. These modes were concluded not to be viable candidates for the mode of vibration that was causing the cracks (Fig. 2). Stress profiles showed the modes were most likely to initiate failures in the third knife-edge— stresses were higher than at either the first or second knife-edge. Since no third knife-edge cracks had been found, and since the cracks had never propagated up to the third knife-edge, these modes could not have been responsible for the cracked seals found in engines.

Laboratory tests of the seal-hub assembly with a loose front snap (made permanently loose by machining) revealed a greater number of vibratory modes reponsive in terms of knife-edge stress per unit vibratory force into the hub. Although the fundamental 2-nodal diam modes had been made less responsive because of being decoupled from the hub, a number of higher frequency modes now became apparent, and 3-, 4-, and 5-nodal diam modes showed a significant response (Fig. 3). There were also fundamental differences in the mode shapes with the loose front snap. The most significant difference is the most obvious—while the front snap had tended to be a vibration node when the hub and seal were in intimate contact, the front of the seal tended to be an antinode when the seal was free, unsnapped from the hub (Fig. 4).

Shaker Results

When the stress profiles of the loose snap modes were analyzed, several matched the characteristics associated with vibration causing cracking. Two, 5-nodal diam modes in particular caused first and second knife-edge stresses much higher than stress at the third knife-edge. Additionally, there was a significant die-out in vibratory stress between the second and third edges.

Part of the shaker testing was a comparison of several hub-seal assemblies that had cracked seals with other high-time assemblies that had not cracked. Statistically, no frequency or mode shape differences which could be attributed to the cracks were found. If the vibration excitation was being turned off in an engine, it apparently was not because of mechanical retuning.

The conclusion from the shaker tests and holography was that cracking occurred because of vibration in one or both of the 5-nodal diam modes which occurred when the front snap is loose. This vibration would have to occur after a rapid engine acceleration to maximum speed when the front snap became transiently loose (Fig. 5). There were no land modes of the correct frequency and no 5-nodal diam for a mechanical coincidence situation to occur.

Core Engine Tests

To assist in defining the mechanism that excited the vibration and to provide a vehicle to demonstrate a repeatable

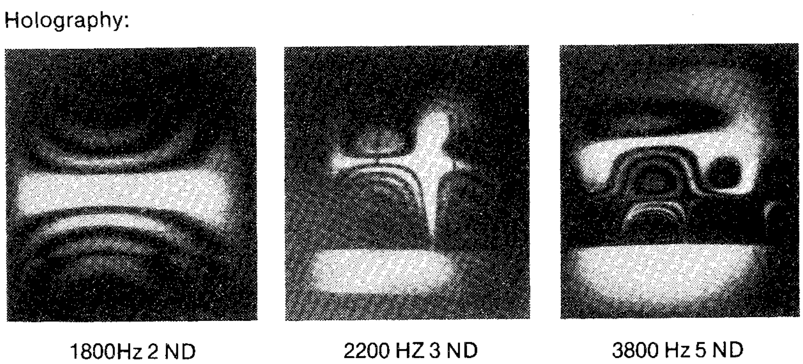

Fig. 4 Lab analysis test holography backs up core engine test and analysis.

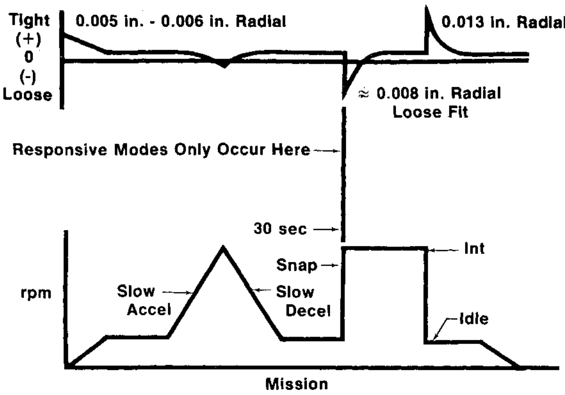

Fig. 5 Front snap fit.

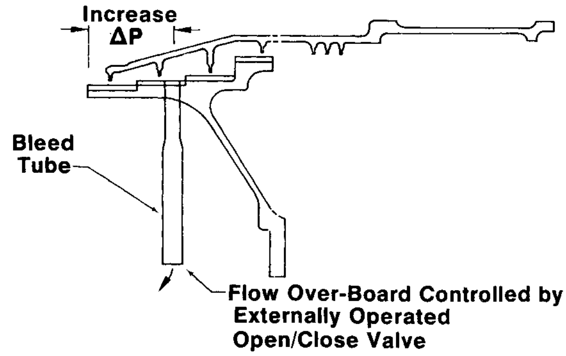

Fig. 6 Seal land bleed tubes amplified instability to confirm predicted ΔP effect.

solution, a core engine was built. The core engine consists of compressor, burner, and compressor drive turbine. The core engine also drove an accessory gearbox and fuel control package. In place of the low spool or fan normally used on an engine, the compressor inlet and turbine discharge were matched to a facility which matched the engine flow, pressure, and temperature conditions over the majority of the flight envelope. The seal hub and land were instrumented with high temperature strain gages. The seal knife-edge gages presented the biggest potential payoff, since there must be significant stress on the knife-edges to duplicate the destructive vibration. The environs of the seal were also instrumented with rotating and nonrotating thermocouples, dynamic pressure pickups, and accelerometers. Wide bandwidth tape recording and realtime spectral analysis were provided.

The shaker tests had already indicated this was a phenomenon that occurred only when the front snap of the seal rotor was loose. This condition is easily simulated in a full engine due to rapid acceleration from idle to maximum rotor speed. The front snap fit was machined open as had been done for shaker tests. The clearance selected resulted in a snap which would be loose at steady-state maximum rotor speed. Normally, the snap would become tight after 30 s, an insufficient length of time for study.

The strain-gaged seal rotor, hub and land were transplants from an engine in which an axial barrel crack had been found. The barrel crack from the second knife-edge had not yet joined the crack from the first knife-edge. During the first operation at maximum speed with the seal environment matching an engine condition, high vibratory stress occurred at a frequency and phase relationship to match one of the laboratory 5-nodal diameter modes. However, after 5 s, the vibration ceased and could not be reestablished at any operating condition. The seal was x-rayed in the engine and it was discovered that the barrel cracks that already existed had joined, splitting the hoop. Subsequent teardown confirmed this and showed that like one other split-hoop seal, it had gouged out its mating land.

This seal was replaced with another that had been found with cracks. These cracks were confined to the knife-edges only and should potentially have a longer test life as the vibration was turned on and off. This seal was not part of a matched set as the previous one had been. The hub and the seal land from the previous tests were not changed in any way.

The second seal would not vibrate. Like the split-hoop seal, it would not respond no matter how extreme the engine operating conditions.

Analytical investigation now had not only the laboratory data but the brief sample of actual seal vibration data. There was also the relevant data from the second seal which would not vibrate. The information collected, the choices of the excitation narrowed:

1) Mechanical vibration—The seal response was not simple mechanical resonance because it did not respond at integral multiples of rotor speed nor fractional multiples. Measured structural vibration did not have a frequency or time history that correlated with the seal vibration. The HCF die-out would not be explained by a mechanical vibration as retuning was not significant.

2) Seal-land coincidence—During the interval the seal was vibrating, vibration of the land was minute and at the incorrect mode shape and wave speed to indicate coincident vibration. This confirmed the expectations from the shaker testing that the land was not involved.

3) Acoustic excitation—This could not be eliminated, but the frequency and mode shape that was excited was not a probable candidate for the acoustic spinning waves or "organ pipe" resonances that were predicted for engine conditions where the vibration occurred.

4) Aeroelastic instability—Although the most abstract, this mechanism became the most likely candidate for the role of the exciting forces.

Analysis of Aeroelastic Instability

An analysis tool in use by Pratt & Whitney Aircraft (P&WA) can predict the relative aeroelastic stability of labyrinth seal land geometries from the parameters of engine operating condition, seal geometry, knife-edge clearances, and vibratory mode shape and frequency (see Appendix). This analysis had predicted the seal would be stable in the fundamental vibration modes (2-nodal diam mode, essentially similar whether the snap is tight or loose). However, the 5-nodal diam modes mapped by holography and analyzed through shell analysis were aeroelasticity unstable for some combinations of knife-edge clearances. A postmortem analysis of the seal that ceased vibrating in the core engine indicated instability in the 5-nodal diameter mode at engine conditions and clearances at which it had been originally built, but would become stable after the clearance had been altered as the land was gouged out by the first knife-edge riding on the split-hoop. (Measurements had shown a radial clearance change of 0.015 in.) Likewise, the second seal which would not vibrate was stable in the gouged-out land.

Additional analysis showed that clearance combinations could be established for the four knife-edges which would be aeroelastically stable for all of the responsive modes detected by holography.

Continuing Core Engine Tests with Instability Stimulation Device

Because stability analysis indicated that one reason the seal did not vibrate was the gouged-out land that occurred after the first test, this land was replaced with a new part which was hand picked to provide the tightest knife-edge clearances. A seal rotor that had been removed from service because of knife-edge cracks was instrumented. The most important feature of this rotor was not that it had previously cracked, but that its clearances, when mated with the new seal land, would be unstable. Stability analysis indicated that the cavity between the first and second knife-edges was especially sensitive to the instability mechanism. The analysis showed as the pressure differential across this cavity was increased statically or dynamically, the entire system tended to become aeroelastically unstable. The mechanisms causing an increased pressure change in real life situations would be clearance changes on the subsequent knife-edges, either static or dynamic. The front to back pressure influence on the first two knife-edges prompted the need for an invention. The invention was necessary to turn on the instability during the next test and then, when the fix was implemented, demonstrate enough margin to handle unpredictables and growth.

The invention is shown in Fig. 6. It is simply a bleed tube pierced into the second cavity, between the second and third knife-edges. By valving this bleed tube to a sink of any desired pressure, the pressure change across the first cavity could be varied almost at will, with the condition most conducive to instability being with the bleed vented to ambient, which increased the normal static pressure drop by 260%.

During the next test of the engine, it was possible to turn on the instability at will, whenever the engine was operating at maximum rotor speed conditions. Once more it was a 5-nodal diam mode, but at a higher frequency than the first seal. Perhaps reflecting a generally tighter set of knife-edge clearances across the seal, the vibratory stresses measured on this configuration were almost twice as high as the short lived seal. Use of the bleed tube invention simulated an additional 30% increase in vibratory stress.

The cause of the seal vibration had been identified. The evidence was even more conclusive that aeroelastic instability was the exciting phenomenon. Dynamic pressure measurements did not detect an acoustic excitation upstream of the seal during seal vibration. Intracavity acoustic spinning waves could not be analytically justified which would match the frequency, engine speed, and air temperature variations. The mechanism was behaving, as the stability analysis indicated it would, in response to the cavity bleed tube.

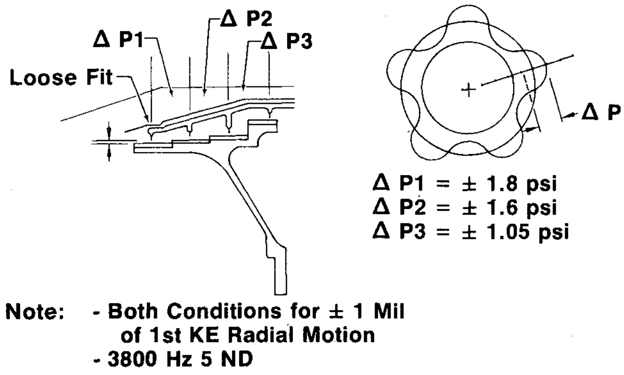

Fig. 7 Bill of material clearance.

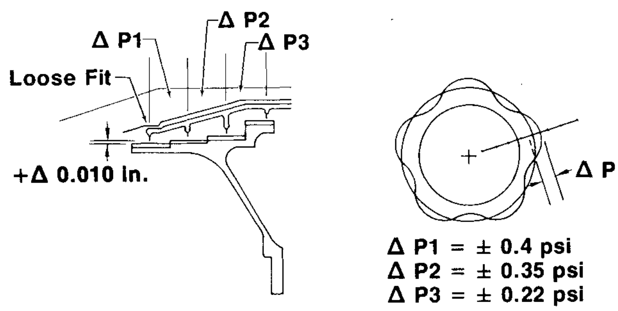

Fig. 8 Increased first knife-edge clearance.

Stability Fix

Having identified the cause of the vibration, a redesign to eliminate the occurrence of aeroelastic instability became necessary. The aeroelastic stability analysis had been pointing the way, as had the experimental evidence from the first seal that ceased vibrating and the second seal that did not vibrate. These were all sending the same message: Adjusting clearances can eliminate aeroelastic instability.

Engine thrust and thrust-specific fuel consumption were very sensitive to flow through this seal. Analysis showed that a large stability margin could be achieved by increasing the clearance at only the first knife-edge while the leakage flow increase was insignificant. For instance, with the four step labyrinth seal in question, the leakage flow was 13 times as sensitive to the fourth knife-edge clearances as to the first knife-edge. A trade could be made that reduced the clearance at the fourth edge by 0.0015 in. against an increase of 0.020 in. at the first knife-edge, while maintaining the same overall seal leakage flow and providing aeroelastic stability throughout the envelope.

Final Core Engine Test

The core engine test bed was easily adapted for proving out the fix by machining abradable material on the first land to open up the clearance over the first knife-edge by 0.020 in. radially. This configuration, which had been such an energetic vibrator before, was now benign with no measurable level of discrete frequency vibration at any engine condition contrived for it. The bleed tube arrangement which had given amplification of the instability prior to the fix could not cause any effect on the vibration with the opened clearance configuration. The aeroelastic instability had been intentionally eliminated for the first time. Inadvertently, the same thing had been done on the previous tests with the gouged-out land.

The final solution for the problem was precisely the configuration tested. Analysis indicated the leakage increase resulting from opening the first knife-edge could be fully compensated by reducing the fourth knife-edge clearance by 0.0015 in. Since the delta in leakage is trivial, this refinement is not really cost effective and was not incorporated.

Fix Addresses All Previous Crack History

A review of the final results explained the physical causes for preliminary evidence and corroborated the initial assumptions:

1) The vibration was in a shell mode that had a stress profile consistent with cracked parts. There were high stresses excited in the first two knife-edges but they were tolerable in the aft knife-edges. Because of the mode shape, the crack did not propagate up to the third knife-edge in HCF.

2) The instability phenomena was self-healing, for once the crack had progressed far enough to split the hoop, the abradable material was gouged out of the front knife-edge, making the system stable. The only remaining cyclic stresses were low cycle fatigue, while long term propagation was slow.

3) The engine conditions during which the instability would occur was common to all engines, essentially any fast acceleration to maximum rotor speed, the typical mission of a fighter engine.

4) A unique combination of parts was required to initiate the instability. Specific clearance combinations across the four knife-edges were required for the system to become unstable.

A postmortem analysis of the population of cracked seals and another group of seals that had accumulated substantial operating time with no cracks indicated a good correlation with aeroelastic stability analysis and the 5-nodal diam mode shape. Because the abradable land could be measured at its virgin dimensions and at the run-in dimensions, two states of the seal stability environment could be examined. With one exception, the cracked seals were indicated by analysis to be potentially unstable in their virgin dimensions. The exception was shown to be stable in the virgin state but unstable with worn dimensions. This was a clear signal.

The fix was reanalyzed to see if a stable configuration was maintained with increased clearances in the aft knife-edges to

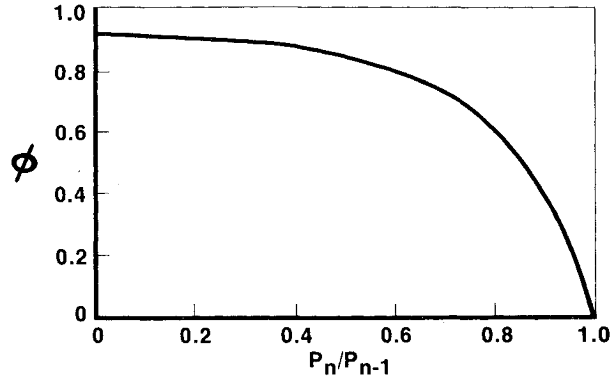

Fig. 9 Flow function vs pressure ratio.

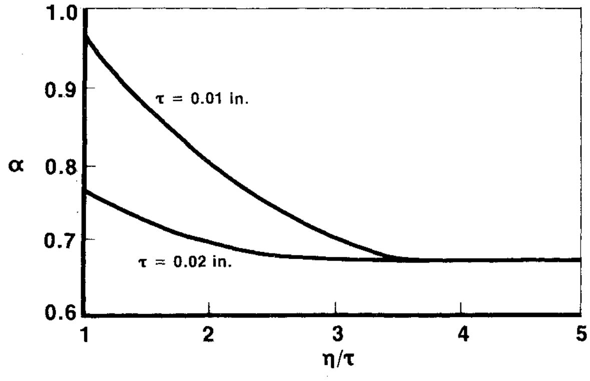

Fig. 10 Flow coefficient vs clearance and lip thickness.

simulate progressive wear. The fix maintained a large stability margin even with clearances representing wear beyond that experienced.

Summary of Design Lessons

The occurrence of excessive vibration within a labyrinth seal in a relatively mature engine design yielded some potentially useful lessons which will be applied to future designs:

1) Avoid inconsistent fixity. Loosening of the front snap was recognized initially but the total method by which it contributed to instability was not seen as a problem from any previous experience. New designs should endeavor to maintain one configuration which can then be analyzed and endurance tested with confidence.

2)Determine the clearance sensitivity of the knife-edges before setting the flowpath. A quasi-steady-state analysis is useful in demonstrating the relative sensitivity of various seal combinations to cavity pressure disturbances. In Fig. 7 the variations in the downstream cavity pressures that result from changing the clearance of the first knife-edge are shown. In Fig. 8 the same clearance variations cause only about 20% of the pressure variation when starting from a new baseline with an increased first knife-edge clearance. Obviously the leakage flow across the seal will also increase in the latter case but only by 4% seal flow (0.1% engine flow). This quasi-steady-state analysis does not constitute an aeroelastic stability analysis, but the results are complementary. This analysis is useful in optimizing the knife-edge clearances, so that for a given leakage flow, the energy fluctuations introduced to the system due to the clearance changes at any of the knife-edges would be minimized.

3) Investigate higher modes. Experience indicated the fundamental vibration mode is not the only worry for stability. Initially, a steady-state clearance sensitivity study should be completed. Ideally this would include a full-scale parametric aeroelastic stability analysis. From this study it should be possible to determine hypothetical axial mode shapes that are most conducive to instability, and then to determine the stability of real modes with similar mode shapes.

4) Evaluate the aerodynamic stability margin at the actual operating clearances. Obviously, this means accurate thermal predictions will be needed for both transient and steady-state conditions. It is also essential to evaluate all of the clearance combinations that can arise from the tolerances permitted.

5) Evaluate the aerodynamic stability margin with deteriorated clearances. After service, the seal clearances are likely to increase as a result of flight maneuver deflections (especially in fighter aircraft), erosion, etc. The stability of these deteriorated seals should be evaluated to determine that the initial stability is maintained for the life of the engine. Do not assume that the seal wear will be uniform from knife-edge to knife-edge, but use rotor-to-case deflection predictions, or service experience if it is available, to establish wear distribution and amount.

6) Verification tests:

a) Shaker tests including holography are a valuable tool to determine the true mode shapes once hardware is available. Since most analytical mode shape calculations cannot determine the relative sensitivity of the various modes, shaker tests are best for this purpose. Recent experience had indicated the most easily excited modes, with a mode shape that is compatible with the aeroelastic seal excitation, are the prime candidates for instability whether or not they are the lowest frequency or least complex modes. All possible fixities should be simulated in the shaker tests.

b) Engine or rig tests that match all of the service environmental features of the seal are the last word in proving a design is stable, but only for the set of clearances tested. Such tests should be with hand-picked components which have the least margin for the design tolerances and/or reflect deteriorated tolerance. Suitable bleed schemes can increase the severity of the test in a reversible fashion to evaluate if there is an excess margin with which to operate.

Appendix

Labyrinth air seals are used in annular passages between rotating and stationary members where an undesirable nominal clearance must be maintained to accommodate tolerance accumulation and off-design operation. Leakage flow through the required clearance is reduced by repeating throttling. Radial steps are provided to disperse the kinetic energy developed at each lip. The resultant pressure ratio across each throttling is appreciably less than the overall pressure ratio, and leakage is thus reduced.

A typical labyrinth consists of a rotating member and a stationary member. If either member vibrates in a mode which includes motion in a direction which changes lip clearance, oscillatory internal cavity pressures will be generated. The resultant seal member vibratory stability may be evaluated by the aeroelastic analysis derived below.

The flow characteristics for any single lip is given in Ref. 7 to be

$$W_n = 2\pi R h \alpha \phi \sqrt{g P_{n-1} / V_{n-1}} \tag{A1}$$

where ϕ and α are given in Figs. 9 and 10. In a multilip labyrinth, the pressure level in each cavity and the steady leakage flow are determinable from the lip flow characteristic and flow continuity.

$$W_n = W_{n+1} \tag{A2}$$

reversible adiabatic expansion,

$$V_n / V_{n-1} = (P_{n-1} / P_n)^{1/\gamma} \tag{A3}$$

and overall pressure ratio,

$$\prod_{n=1}^{N} P_n/P_{n-1} = P_d/P_u \tag{A4}$$

The oscillatory flow past a single lip may be defined in a quasisteady fashion by expanding the single lip flow characteristic equation:

$$(W+\Delta W) = 2\pi R(h+\Delta h)(\alpha+\Delta\alpha)(\phi+\Delta\phi)$$
$$\times\sqrt{g(P+\Delta P)/(V+\Delta V)} \tag{A5}$$

where

$$\Delta\alpha = K\alpha\Delta h/\tau$$
$$\Delta\phi = K\phi[(P_n+\Delta P_n)/(P_{n-1}+\Delta P_{n-1}) - P_n/P_{n-1}] \tag{A6}$$

and

$$(V+\Delta V)/V = [P/(P+\Delta P)]^{1/\gamma} \tag{A7}$$

Substitution and linearization reduce the single lip oscillatory flow characteristics to

$$\frac{\Delta W}{W} - \left[\tfrac{1}{2}\left(1+\frac{1}{\gamma} - \frac{K\phi P_n}{\phi P_{n-1}}\right]\frac{\Delta P_{n-1}}{P_{n-1}} - \frac{K\phi\Delta P_n}{\phi P_{n-1}}$$
$$= \left(\frac{1}{h} + \frac{K\alpha}{\alpha\tau}\right)\Delta h \tag{A8}$$

The behavior of the fluid in each cavity can be examined to relate unsteady pressure and through-flow. A cavity is visualized to be a torus filled with fluid moving in the direction of motion of the rotating seal member at a determinable steady velocity. The unsteady characteristics of the fluid are most readily defined by an observer who is moving with the fluid in the channel. From this viewpoint, the frequency of vibration associated with a traveling deformation wave is

$$\omega = \omega s \pm E/R(U-\Omega R) \tag{A9}$$

where the sign distinguishes between backward and forward directions of wave travel in the vibrating structural member.

Newton's second law is applicable to a pressure wave in the fluid filled toroidal cavity,

$$A\frac{\partial\Delta P}{\partial X} = \rho A\frac{\partial^2\Delta x}{\partial t^2} \tag{A10}$$

The rate of change of density of the fluid in the channel is the combined result of expansion in the circumferential direction and local net unsteady flow into the channel. This may be expressed in terms of proportional changes,

$$\frac{1}{\rho}\frac{\partial\Delta\rho}{\partial t} = \frac{\partial^2\Delta X}{\partial x\partial t} + \frac{(\Delta W_{n+1} - \Delta W_n}{2\pi R\rho A g} \tag{A11}$$

The traveling wave is harmonic in space and time so frequency may be substituted for differentiation with respect to time and (wave length)$^{-1}$ for differentiation with respect to circumferential distance. A 90-deg phase shift is also introduced at each differentiation. The resultant cavity equations may be combined:

$$\pm\left[i\frac{\omega}{\gamma P} - i\frac{E^2}{\rho\omega^2R^2}\right]\Delta P - \frac{(\Delta W_{n+1} - \Delta W_n)}{2\pi R\rho A} = 0 \tag{A12}$$

where the sign distinguishes between backward and forward direction of wave travel in the cavity fluid.

Oscillatory pressures external to the labyrinth are considered to be insignificant.

$$\Delta Pu = \Delta Pd = 0 \tag{A13}$$

The stability of any natural mode of vibration of either structural member may be evaluated for forward or backward wave travel. The particular mode shape defines the distribution of unsteady lip clearance. The N lip equations and $N-1$ cavity equation may be solved simultaneously to determine the magnitude and phase of the unsteady lip flows and the unsteady cavity pressures. Vibratory stability is controlled by the energy transfer from the fluid in the cavities to the vibrating structure. This can be determined by evaluating the integrated product of unsteady cavity pressure and exposed cavity wall normal velocity (from natural mode of vibration of the structural member). The overall stability of a given seal configuration is characterized by its least stable mode, and the least stable modes of various configurations provide a basis for selecting an improved design.

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